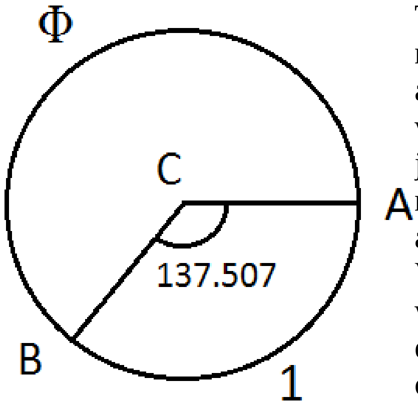
How do growth patterns in nature cause the appearance of the Golden Ratio?

The natural world is full of many mysteries, some we attempt to explain, others we just accept and ignore yet, as Albert Einstein once said, 'Look deep into nature, and then you will understand everything better.' The golden ratio is one of these mysteries. It is a mathematical anomaly that we continuously observe, mostly without acknowledgement.
So what is the golden ratio? It is an irrational number, of value 1.6180...[1] that as far as we can calculate has no finite value. It is identified in the Greek alphabet as phi or Φ. There are three methods to calculate it. It was originally discovered by Leonardo of Pisa who is better known as Fibonacci, "son of Bonaccio"[2]. He did not directly calculate Φ but he simply came up with a geometric sequence that for every successive term the ratio between them became closer and closer to the golden ratio.
The Fibonacci sequence was developed from an idea that Leonardo wrote[3]. The original idea for the sequence was 'How many pairs of rabbits will be produced in a year, beginning with a single pair, if in every month each pair bears a new pair which becomes productive from the second month on?'[4]. Explaining this using algebra we have $u_n = u_{n-1} + u_{n-2}$ so each term after the first two is the sum of the two terms before it. Numerically this sequence is 1,1,2,3,5,8,13,21,34,55... now to find the golden ratio from this you can divide the last number by the one before it, the further into the sequence you are the closer to the golden ratio this number will be. For example if you divide 55 by 34 you get 1.6176...[5] which is only 0.0004 away from the real value. Once this irrational value was discovered mathematicians worked out new methods to calculate it to a greater degree of accuracy. The first of the two modern methods is also a numeric sequence. $\sqrt{(1+\sqrt{(1+\sqrt{(1+\sqrt{(1+\sqrt{(1+....)}}}}}$ this sequence works by becoming more accurate every iteration you perform. This is a simple way to calculate the rational approximates of the ratio using a scientific calculator.
However, there is an even simpler equation that equals the exact value of the golden ratio. This was developed by Euclid using a straight line quadratic equation that simplifies to this: $1/2(1+\sqrt{(5)}) = \Phi$. This was derived by algebraically simplifying the previous method with the knowledge that it tends towards Φ.



The value of the golden ratio is not the only reason it relates to nature, the golden angle (α) appears more commonly than the actual value of phi itself. To find α we imagine a circle (shown left) where an arc of length 1 subtends the circle between two lines A B joining at the centre C to form a line segment ACB, now if the remaining arc of the circle is of length phi then the angle ACB is approximately $\alpha \approx 137.507^o$.
With the maths behind Φ explained we can move onto the more wondrous side of the golden ratio; how it manages to perforate our existence without any simple explanation. This irrational value often turns up where ever maths and biology collide; many speculate as to whether this is pure coincidence or whether there is an actual correlation between the appearances. Nevertheless that is not what this project is designed to answer. Rather we shall look at why it appears where it does, looking at what causes plants to grow in such a way that it follows the golden ratio, attempt to understand why certain shells grow to the proportions of the golden ratio and delve into other perplexities that surround the number Φ.
This essay shall be built around five examples arranged into four different sections with similar examples being joined together. From my research these examples seemed to have the strongest

1 Golden ratio quoted to 12 significant figures: 1.61803398875... it has been calculated accurately to several million digits.
2 Information obtained from C.B.Boyer (1985) A History of Mathematics (First Princeton Paperback printing).
3 Orignally written in *'Liber abaci'* that was completed in 1202 by Leonardo of Pisa.
4 Translated indirectly from the text *'Liber abaci'* as no English versions of the text are available.
5 1.61764705882 to 12 significant figures.

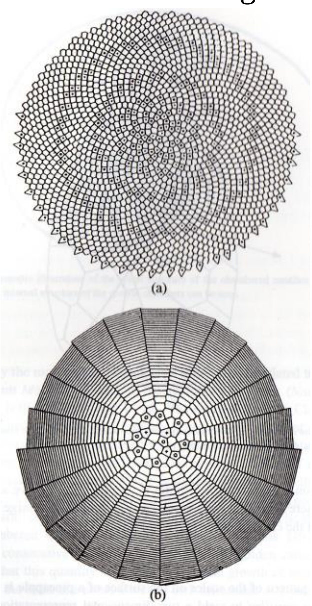
relation to the golden ratio. I have discounted many others where the link seemed tenuous or unlikely. One example that I discounted is based around symmetry[6]. This attributes the two dimensional fivefold symmetry and three dimensional icosahedral symmetry of plants and animals from the phylum *Echinodermata.* However this type of symmetry is called material symmetry as it is approximate and is affected by the imperfect growth of the organism. Although I agree that the symmetry does occur and can be related to the golden ratio there seems to be no explanation using the growth patterns of the plants for why it has appeared.

<u>Sunflowers</u>

The first example that we shall analyse is that of the pattern of seeds inside a sunflowers seed head. The spiral patterns inside its seed head are incredible feats of biology and mathematics. Contained inside that simple flower are two spirals, one clockwise, one anticlockwise, each independent of the other but both formed using the same growth pattern of these seeds. What are the chances of these patterns forming randomly within nature, miniscule! However they do not form randomly, the mathematics of the golden ratio guides its growth.

In almost all sunflowers the number of seeds in each spiral is a Fibonacci number, they form a ratio of the clockwise to anticlockwise spirals of 89/55 or 144/89 for larger sunflowers. This immediately indicates the golden ratio as both of these numbers are part of the Fibonacci sequence, however it is impressive how important that really is and both I.Stewart[7] and R.A.Dunlap[8] show this using different computer simulations but with both achieving the same effect.

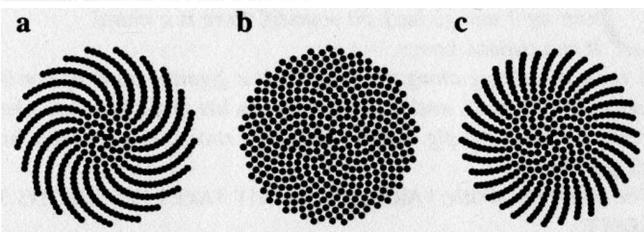
R.A.Dunlap's computer simulated growth pattern, shown left, uses two different growth angles, both related to the golden ratio, however one of these uses a rational approximation of the golden ratio. This shows just how accurate sunflowers growth patterns have to be for this pattern to appear.



The upper image (a) is formed by using a growth angle of $2\pi/\Phi$ radians[9] between the seed heads, this means that every next seed head will appear at $2\pi/\Phi$ radians from the next. As you can see in the simulation it forms an image very similar to those produced by actual sunflowers.

The second image (b) is created using a growth angle of $2\pi/(21/13)$ radians, the eighth rational approximate of Φ[10], clearly you can see that it does not form a pattern that best uses the space inside a seed head. It instead forms 21 radial arms, for any rational approximate of the ratio used in this growth pattern simulation, the denominator of the value is the number of radial arms formed. So for it to produce a pattern similar to the upper image the denominator of the rational approximate must be greater in value than the number of seeds around the edge of the head.

I.Stewarts model shown below has three different simulations, the middle one is using the golden angle of 137.5° as the divergence angle between



the seed heads, this clearly produces both spirals inside the seed head. The left simulation uses a growth angle of 137.3° and only produces the clockwise spiral and the right simulation uses a growth angle of 137.6° and

6 Information obtained from R. A. Dunlap (1997) - The Golden Ratio and Fibonacci Numbers. (World Sci. Pub. Co., NJ)

7 Image and information obtained from I.Stewart (2005) The Magical Maze – Seeing the world through mathematical eyes (Phoenix, Orion Books Ltd, London).

8 Image and information obtained from R. A. Dunlap (1997) - The Golden Ratio and Fibonacci Numbers. (World Sci. Pub. Co., NJ).

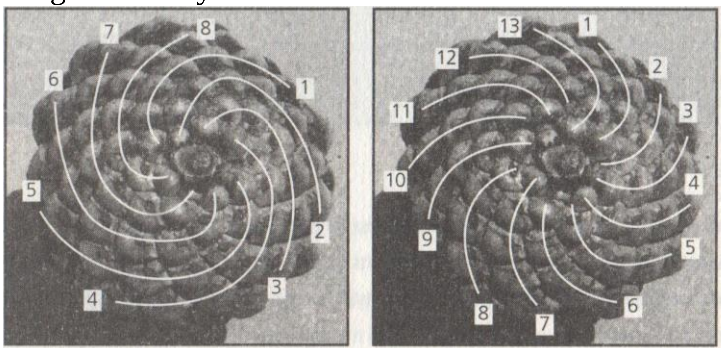
9 $2\pi/\Phi$ radians = 222.492° = 360° - α

10 $2\pi/(21/13)$ radians = 222.857°

produces only the anticlockwise spiral, this slight change of 0.3° and 0.1° makes a massive difference in the packing of the seeds inside the seed head. The effect the packing of these seeds has on the plant itself is enormous, if the seed head had only 21 radial arms, as in R.A.Dunlap's simulation then the number of seeds contained inside the same space as a regular sunflower seed head will be dramatically reduced. Looking at this from an evolutionary perspective it is obvious how having fewer seeds in the same space will cause that plant to be less competitive. However we will come back to that later; firstly it is important to see how the plant grows the seed head.
Starting at the beginning of the biological process of a plants growth we must look at its genome. Inside this genome there will be coding for the proteins that are synthesised in a specific order that produces cells which will grow into which ever type the plant requires. These cells then grow into the space where they are created. When a plant is growing it is never just one cell at a time, there will be hundreds of cells attempting to grow along the same stem. Plant cells have cellulose cell walls that are very strong and rigid, this means that they cannot be moulded into whatever space surrounds them, they must pack together in a very specific order. The method used by plants to minimise this issue is by using primordia. Primordia are groups of simple cells called primordial cells that are capable of beginning the growth of a new organ.
The primordia form as the apex of the plant growth moves away. Consider it as a moving factory, at the tip of the growing stem, the apex, the primordium is just being created, it is placed on top of the previously placed primordia. As the primordium continues to grow and increase in size it pushes the apex higher where this then produces more primordia that are placed on top of the original primordium and the process begins again. So each primordium is forced into the biggest area available on top of the other primordia.
Every single primordium can grow into a seed, or a leaf, or a petal depending on the arrangement of proteins inside the primordial cells. This is a very important concept when looking at how the seed head grows, the newest seeds are at the centre of the seed head as this is the apex and not at the edge; if you have ever looked into the centre of a sunflower seed head it is incredibly tightly packed as the small new seeds are formed and push away the larger older seeds.
Now going back to how this is important to the evolution of the plant, a sunflower seed head that is not densely packed will be weak and will not be using the space efficiently. For a seed head to be efficient every part of the space available must be filled with seeds. If the seed head contained only 21 radial spokes of seeds the waste of energy will be massive and the number of seeds that go on to generate from the original plant will be reduced compared to a seed head where it's entire seed head is full of seeds. Due to natural selection the seed head with the most effective packing will survive and pass on its genes and the sunflower which does not have efficient packing will not survive as well and so its genes will not be passed on.
So over the course of the evolution of the sunflower the golden angle has been chosen as the best divergence angle for the primordia. What reason has evolution chosen this angle? Well when looking at the simulations we have seen that using any rational approximation will produce radial spokes; whereas the golden angle goes the complete opposite way, forming an angle furthest away from the rational approximations and so causes the densest packing possible for the seed head.
This example has shown us that the golden angle is produced by the necessity for the plant to outcompete other sunflowers, evolution has driven it to the most efficient packing method and so the dense packing growth pattern of the sunflower has caused the appearance of the golden ratio.

<u>Pinecones and pineapples</u>
The next example of where the golden ratio appears in nature is in pinecones and pineapples, I have grouped these two examples together as they are very similar in their appearance and cause. These examples also have similarities with the sunflower seed heads as they all use primordia to grow these seed capsules. The patterns on pinecones and pineapples are much more difficult to find compared to those on sunflowers as they are three dimensional. I shall first talk about pinecones, the golden ratio can be found on pinecones by looking at them from the underneath. Viewed from underneath, it is possible to identify two spirals running through the pinecones scales, one spiral can

be seen in the clockwise direction, the other in the anticlockwise direction. The number of clockwise and anticlockwise spirals is usually successive Fibonacci numbers, for example in the image below[11] you can see that there are 8 anticlockwise spirals and 13 clockwise spirals. The



appearance of these consecutive Fibonacci numbers immediately indicates that the golden ratio appears in some form during the growth of pinecones.
The appearance of these spirals was first noted by a German Naturalist Karl Schimper and a French crystallographer Auguste Bravais in the nineteenth century. They each produced separate studies on the factors governing patterns of phyllotaxis and they both identified pinecones as an example of Fibonacci numbers appearing in their growth.
In 1968 an American mathematician Alfred Brousseau decided to perform a study of 4,290 pinecones including ten different species of Californian pine tree[12]. On completion of the study only 74 out of 4,290 pinecones had not followed the Fibonacci sequence, 98.3% had. However the scientific community did not except this as it was believed a too small sample group.
In 1992 the experiment was repeated by a Canadian botanist R.V.Jean[13] where he increased the sample size to 12,750 pinecones including 650 different species. This time the study showed that 92% of the cases followed the Fibonacci sequence. This means that only 1,020 out of 12,750 did not adhere to the pattern, for me this is proof that this pattern does not appear randomly. If this does not occur randomly then we must assume that the pine trees have evolved into this growth pattern as it improves their chances of survival.
Pinecones are either male or female, the male cones contain pollen sacs that are released into the air when wind blows through the open cones, whereas the female cones contain ovules that when fertilised by the pollen from the male cones become seeds. The female pinecone is designed to protect the valuable seeds from damage that could be caused by rain, insects or birds and still be able to receive the pollen from the male cone, whilst the male pinecone is designed to protect and then release the pollen.
Subsequently for this structure to be well adapted it must be strong and contain the most pollen or ovules that it can, this sounds very similar to the adaptations required by the sunflower seed head, so once again the golden ratio has appeared when dense packing is required.
Pinecones also grow using primordia where the stem that is attached to the branch of the pine tree is the apex. This means that the apex is not pushed upwards but rather the primordia are pushed down as they grow, once again the primordia are forced into the largest space on top of the previously placed primordia; this produces the tear drop shape of the cone and causes the divergence angle of approximately the golden angle. The precision of the divergence angle on the pinecones can be much smaller than that of sunflowers as the number of scales on a pinecone is only in the order of tens whereas the number of seeds inside a sunflower seed head is in the order of hundreds.
In the previous section I discussed how when using a rational approximate of the golden ratio it produced a number of radial spirals equal to the denominator of the approximate; as the number of spirals on a pinecone is, on average 13 clockwise, the denominator only has to be 13 to produce the same spiral shape. Although the growth angle is not as precise as in sunflowers it is still chosen for the dense packing it produces on the cone.

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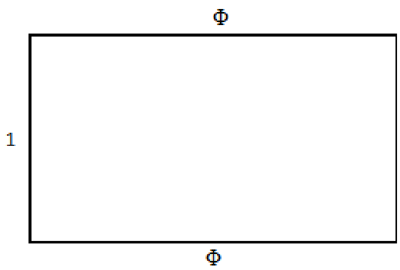
[11] Image obtained from F.Corbalan (2010) The Golden Ratio – The beautiful language of mathematics (RBA Coleeccionables, S.A.)

[12] Research data described in A.Brousseau (1968) Fibonacci Quarterly – On the Trail of the California Pine (St. Mary's College, Calif)

[13] Research data described in R.V.Jean (1994) Phyllotaxis : A Systemic Study in Plant Morphogenesis (Cambridge University Press)

Next we shall look at pineapples. Pineapples were originally named for their similarity in appearance to the pinecone, they also have scales that form a pattern with two spirals, one clockwise and one anticlockwise. The similarities do not end there, the number of spirals on a pineapple is eight rows spiralling to the right and thirteen spiralling to the left[14]. Once again consecutive Fibonacci numbers are appearing.
Pineapples are also seed containers, they begin their growth as flowers, which once pollinated will grow into a single large fruit. This fruit contains starch storage and the seeds of the plant. For the pineapple to be effective at producing more plants it needs a fruit that is protected on the outside from damage and disease. To protect the seeds and starch inside the pineapple, the scales on the outside of the fruit are produced, these were originally the flowers of the plant that have died and formed strong scales in a pattern that covers the entire outside of the fruit.
So for this example we are not looking at the growth of seeds but rather the growth of flowers that then die and harden to form scales. However this does not really affect what we are looking at because flowers grow from primordia in the same way that seeds do, there will just be slightly different cells inside the primordia for the flowers. To protect the pineapple effectively the scales must be packed together tightly with no gaps between them, if gaps were formed then the fruit would quickly rot. To do this the flowers must be grown in a certain pattern. Unsurprisingly as dense packing is required the golden ratio governs this pattern, with the flowers being grown with a divergence angle of the golden angle.
So once again the golden angle appears as primordia are required to grow into the strongest and most efficient pattern that they can. There appears to be a pattern forming here; however, these past three examples have been very similar shapes requiring very similar criteria for their effective growth. The next examples differ from this.

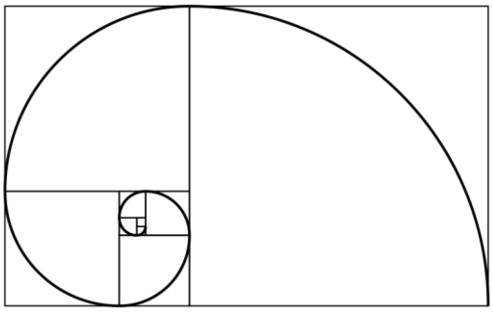
<u>Conical shells</u>
The golden ratio has another unique property that we have yet to discuss, golden rectangles. This is another geometric application of the golden ratio that has been studied in great detail for many reasons, one being its attractive properties, many great artists and architects such as Leonardo da Vinci incorporated the golden rectangle into their art and constructions without understanding the mathematics behind it. The golden rectangle is also able to produce a logarithmic spiral; this logarithmic spiral forms the same shape as that of some conical shells.
During my early research of this section I found much dissent on the internet about whether this did actually occur in real life or whether it was just a rumour, however once I researched into greater detail about it, although I could not find any direct field research on it, every book that I looked at talked about the relationship between the two being exact; therefore I decided to still include this example.
Firstly I should explain what a golden rectangle is, as there are many methods of creating them, I shall use the simplest method I have found. Quite simply it is a rectangle which has sides of the ratio 1 : Φ. The rectangle produced will not be anything special, it will appear to be a regular rectangle, however, if you remove a square inside the rectangle of the same length as the shorter side with one side of the square touching the shorter side of the rectangle, the shape you will be left with will be another rectangle with sides to the ratio of 1 : Φ. I have illustrated this below, as you can see the rectangle we are left with has sides of Φ-1 and 1. The more mathematical of you will have realised that this should no longer be a 1 : Φ ratio, however it is as Φ-1 = 1/ Φ.

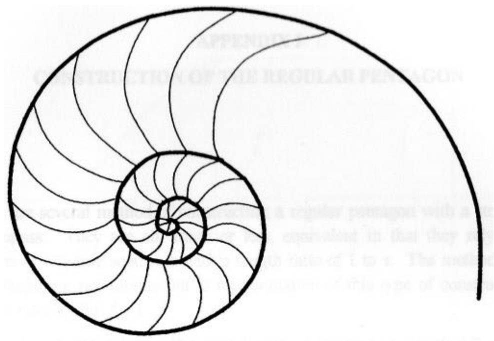


The reason for that mathematical

[14] Information obtained from F.Corbalan (2010) The Golden Ratio – The beautiful language of mathematics (RBA Coleeccionables, S.A.)

anomaly surpasses my understanding so we shall just accept that it happens.
Now if we continue to remove squares from each new rectangle formed, we produce an image from which we can find a logarithmic spiral. To do this we draw a curve from the corner of the smallest square to its opposite corner, then from the adjacent corner on the larger square to its opposite corner. Repeating this for all the squares inside the original golden rectangle, you produce a spiral that increases its growth rate by a factor of Φ every time. An image of this is shown below[15].





This image is strikingly similar to cross section of some conical shells. The most exact example is that of the chambered nautilus[16].
The chambered nautilus builds its shell by constructing larger and larger chambers on top of the previous chambers which it proceeds to seal off. The sealed off chambers are then used to regulate buoyancy. This sort of growth is called self-scaling, the tendency to grow in size but maintain the original shape, it is common of many shells. A cross section of the nautilus is shown below[17] from these two images it is easy to see how they are almost exactly the same shape.
For the nautilus to survive it must have a shell that is large enough to protect its body as it grows, it must have a shell that is strong enough to protect against predators and it must have a shell that is easy and efficient to construct. The shell must also grow at the same rate as the creature living inside it, otherwise the shell would be too large to carry or too small for it to fit in. The chambered nautilus is an ancient animal, fossilised examples of this creature have been found to date back to the early Pleistocene era[18]. For it to have survived for such an immense period of time without dramatic change to its design we can be sure that it is evolved to survive in its habitat perfectly.
When a shell grows it does so by adding cells onto the edge of the previously laid cells. Growing in this manner enables the shell to easily repeat the same pattern over and over without changing the shape of the shell at all, only increasing its size. These shells are strong because they are made from the same material that our bones are made from; they are the exoskeleton of the animal that lives within them. The genes of all the different species of shelled animals will code for the different growth patterns that they use to produce their specific shells, this brings us to the question of why the chambered nautilus' genes coded for it to grow into a logarithmic spiral with an increasing growth rate of Φ.
To explain this we must look to the geometric strength of a spiral. Circles and arches are used in architecture due to their high tensile strength as the physics of these shapes causes the force to be spread evenly over the whole shape. The same can be said for spirals, although slightly weaker than a circle or an arch, their tensile strength is still very high, this is vital for an organism that will be living in areas of massive pressure as without the protection from the pressure the soft body of the chambered nautilus would be crushed. If a human was to dive to 2,000 feet deep, the same depth as the chambered nautilus, then terminal tissue damage would occur and we would die; whereas the chambered nautilus is protected by a spiral shaped exoskeleton that is able to withstand the pressure easily.
The shell also needs to be able to grow with the organism. The spiral is a logical choice here again. I mentioned earlier the term self-scaling, where an object grows in size but maintains its shape, the

15 Image obtained from Wikimedia Commons (http://en.wikipedia.org/wiki/File:Fibonacci_spiral_34.svg)
16 *Nautilus pompilius*
17 Image obtained from R. A. Dunlap (1997) - The Golden Ratio and Fibonacci Numbers. (World Sci. Pub. Co., NJ)
18 2.588 million years ago

growth of a spiral will always be self-scaling. The growth of the organism inside is also self-scaling, the chambered nautilus' body shape does not change as it grows, only its size increases, and therefore it requires a shell that will also do this. These two reasons are why I believe a spiral shape was chosen by the evolution of the chambered nautilus as the best design of its shell.
However, we still have not answered why the golden ratio is used as the growth factor. The answer to this is simple; this matches the exact same rate at which the chambered nautilus grows inside the shell. As the nautilus grows it creates a new chamber to move into and it seals off the previous chamber, each new chamber's volume is larger by a factor of Φ, so in the time it takes for the nautilus to outgrow its old chamber it must have had time to build the next one. The value of Φ appears because this is the perfect value for this relationship; it produces a new chamber that is large enough to contain the nautilus in enough time for it to move into it before it gets too large for the previous chamber.
To conclude this section, the golden ratio appears inside the chambered nautilus' shell as the growth rate factor, it appears here because this is the perfect value between the relationship of time needed to create the next chamber and time taken for the nautilus to outgrow its previous chamber.

<u>Leaves on stems</u>
The final example we shall look out is the position of leaves up a stem. As a stem grows upwards it shoots out leaves at certain points. If you view these stems from above it can be seen that the angle between these leaves is once again the golden angle. In 1914 T.A.Cook, a very famous mathematician who did a large amount of research into the spirals of nature, wrote "the fact that plants express their leaf arrangement in terms of Fibonacci numbers, so frequently that is passes for the normal case, is proof that they are aiming at the utilisation of the Fibonacci angle…"[19]. I do not quite agree with a plant aiming for something, but I can understand that the plants genetics have been coded in such a way that it grows towards this angle, we shall now look at what reason the genes have adapted to do this.
When a plant is growing it will be attempting to get the most light possible to its leaves, this is why the leaves grow away from the stem and why leaves are flat, wide and face the sun. The plant will also attempt to outgrow its rival plants by either growing higher and out of the competition of other leaves, or by growing wide and bushy in an attempt to smother other plants it is competing with. However none of this would matter if its own leaves blocked each other. For instance, imagine that all the leaves on a plants stem grow in a strict rational pattern where a leaf appears every five centimetres up the stem and it is 90° across from the previous leaf. After four new leaves the rotation would be complete and the next leaves will begin growing above the previous ones, this will block out the light reducing their ability to photosynthesise; or another example would be to say instead that every new leaf grew 10° around the stem from the previous leaf, this would then take 36 leaves before they started to grow above each other, however you would have a very off balance plant, halfway through its growth you would have 18 leaves on one side and none on the other. These were obviously very simplistic examples; however they demonstrate that, these are not viable growth patterns for an effective plant.
The angle used is, in a remarkable number of cases, the golden angle. Each new leaf appears 137.507° around the stem from the previous leaf. Going back to T.A.Cook's quote he finishes the sentence by saying "…which will give minimum superposition and maximum exposure to their assimilating members." Once again the Golden angle produces the perfect angle, it allows for the largest number of leaves to be produced without causing them to overlap. What causes the golden ratio to be the best number for this? Well it causes this because the golden ratio is the best irrational number, it is more irrational than other irrational numbers because it is the hardest to approximate by rational numbers[20]. The measure of how irrational number is can be determined by how fast the

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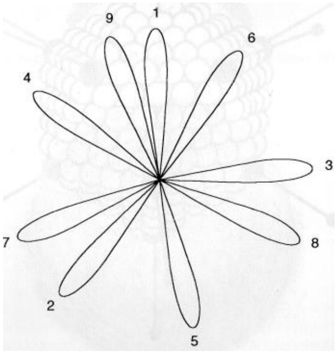
[19] Information obtained from: T.A.Cook (1914) The Curves of Life : Being an Account of Spiral Formations and Their Applicattions to Growth in Nature, to Science and to Art (Dover publications INC, NY)
[20] Information obtained fom J.A.Adam (2009) A Mathematical Nature Walk (Princeton University Press, N.J.)

value of the errors between the rational approximates goes to zero. It is a proven mathematical fact that the errors shrink slower for the golden ratio than for any other irrational number.



This means that it is the least likely number, out of all other possible combinations, to produce leaves that will overlap. Therefore it is clearly the best choice for plants to stop their leaves from overlapping; an image[21] is shown to the left of a plants stem using the golden angle as the growth angle between the leaves. From this image you can see that so far no leaves have overlapped at all and there is still plenty of space between the leaves, following this projection on it takes nearly 30 leaves before all the space is filled and slight overlapping begins.
Concluding this example we can see that plants have been using the golden angle as the growth angle between their leaves as it is mathematically proven as the best value to do so. No other number will make better use of the space and no number will make the leaf arrangement more efficient, so over time the plants have adapted to use the most effective angle possible.

Conclusion

We have now looked at five examples of where the golden ratio and growth patterns in nature collide. We have looked deep into the mechanics of how the golden ratio can be found and attempted to relate how the organism has adapted to use this value and the reasons for its choice of this very specific value.
To summarise these examples, we begun with sunflower seed heads; these seed heads used the golden angle as the divergence angles between their seeds. Using the golden angle produced the most efficient packing possible creating the strongest and densest seed head. Next we looked at pinecones and pineapples; pinecones and pineapples use the golden angle as the divergence angles between their scales. Once again the golden angle produced the strongest cones and scales. The next example was slightly different with the golden rectangle being used to produce a logarithmic spiral that matched the shape of the shell of the chambered nautilus. In this example the shape was chosen because it was strong and the growth rate of factor phi was used as it matches the growth rate of the organism inside the shell. Finally the leaves on stems use the golden angle as the divergence angle between the leaves and plants have adapted to this number because it is the most irrational number possible and so will enable the most number of leaves to be grown without them overlapping.
All these examples have their unique differences, but they are all related by the fact they use the golden ratio in some form to assist their growth. For some reason this value keeps being chosen by nature as the most effective way to grow for organisms. After looking through all these examples I believe that there is a single simple reason for this and I brushed on it in the final example, the fact that the golden ratio is the single most irrational number in the entirety of mathematics.
Nature is irrational, through to its core nature is full of irrational numbers, nature tends towards chaos, the entirety of the universe tends to chaos, this is the second law of thermodynamics. It is an unstoppable law of physics that chaos will be chosen over order. It is therefore not surprising that the most chaotic number possible, the golden ratio, appears at the heart of nature.

[21] Image obtained from R. A. Dunlap (1997) - The Golden Ratio and Fibonacci Numbers. (World Sci. Pub. Co., NJ)

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